Center for Algebraic Thinking

MODULE

Modeling: Physical Representations

BACKGROUND
This module is focused on the teaching of algebraic modeling of quantitative relationships observed in physical representations. By "physical representation," we mean any physical event that may be observed and/or manipulated to represent quantitative relationships "in the world." These events are "real world" in so far as they may be experienced in the world. Their relevance to student lives may occur only in the context of the experience generated to create learning in or out of the classroom that is motivated by the sense-making that comes from interpretation, problem-solving, or the engagement through active participation. The skills developed to model physical events are critical to study in science, engineering, economics, social science, business, construction, etc.

What's the important math?

The power of algebra is that it may be used to model quantitative relationships represented in the observed world. Students advance their understanding of both algebra and the physical world by learning to:

- make careful observations of physical systems,
- identify quantities of relevance,
- understand patterns of relationships among these quantities, and
- modeling these relationships with algebraic equations, graphs, and tables of data.

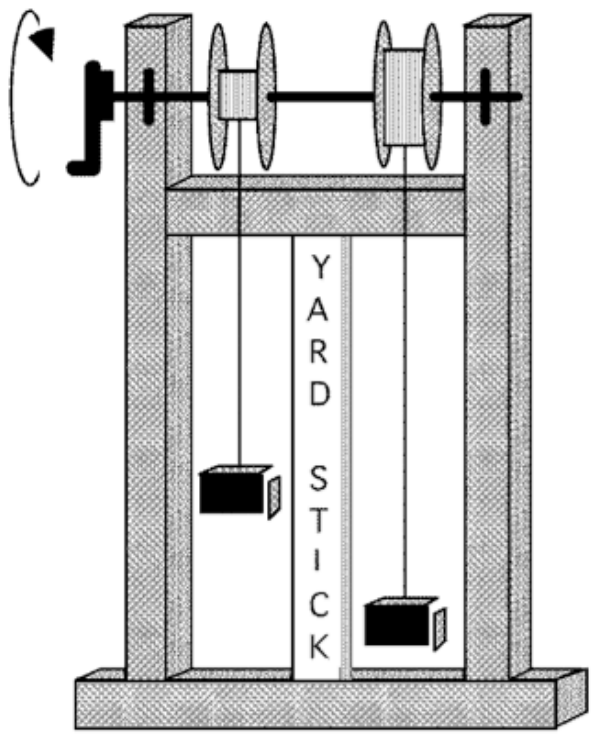
The equations and graphs represent two very clever tools for answering quantitative questions that may be used to predict physical events from observed rules of physical behavior. The utility of algebra depends on the validity of the algebraic models and the reliability of their interpretations based upon a) an understanding of the meaning of the symbols and graphical representations used, and b) the correct application of the rules and procedures used in the "reduction and comparison" of algebraic expressions in equations and the rules for representation and transformation of graphs.

1) <u>SET</u>: **Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers' prior knowledge)**
[note; it would be helpful to have the apparatus below built for classroom use than to use the illustration only]

Consider a physical situation of a crank that winds up two spools of string with weights attached. The spools are each 5-inch circumference and 3-inch circumference, and the beginning locations of the weights differ initially. Develop algebraic expressions and equations that may be used to model answers to the following questions about when and where the weights will be after some number of cranks from 1 to n.

1. "Predict the distance between the weights after an arbitrary number of cranks.
2. Determine whether and, if so, when one weight will ever be twice as high as the other.
3. Determine whether and, if so, when the weights will meet at the same height." (p. 90)

Fig. 1 The winch. From "Inscribing the Winch: Mechanisms by Which Students Develop Knowledge Structures for Representing the Physical World with Algebra" by A. Izsák (2000), *The Journal of the Learning Sciences*, 9, p. 33. Copyright 2000 by Lawrence Erlbaum Associates, Inc. Reprinted with permission, http://www.informaworld.com



2) <u>STUDENTS</u>: **Watch video clips of students describing their thinking as they engage with problems**
What do you learn from what you are hearing or seeing regarding students' thinking?

Interview Questions for Physical Representations and Algebra:
1. What do you observe happening to the weights as the crank turns?
2. What causes this reaction?
3. How could you describe what happens mathematically?
4. Can you write an equation for the relationship between the heights of the weights as the crank turns?

Physical Representation Problems: [Note: These problems should be demonstrated to the research subject one at a time with the interviewer holding the matchbox cars. The students should be asked to write whatever equations, pictures, or expressions, tables, graphs, etc. with a permanent ink sharpie on a large flipchart. As pages get used up, they should be numbered, ripped from the pad and taped to a wall for students to refer to as needed. Newer entries should be written large and below the earlier ones in a clear succession. Students should not be permitted to erase or obliterate their work, but instead, they should place an 'X' next to any work that they think is mistaken or that they would like to delete. Prompt students to share their thinking about their choices as they work.]

Using toy matchbox cars, show two cars traveling in the following scenarios.

1. Show one car moving at constant speed across a table by holding the car with your hand. Ask the subject to draw a distance versus time graph of the motion.

2. Add another car to the same scenario, but with a slightly greater constant speed. Ask the subject to draw a graph with the motion of both cars represented.

3. Create a scenario where one car has a few second headstart from the second car and travels at a constant speed, but the second car travels at a higher speed than the first. Ask the subject to graph the distance versus time for these cars.

4. Start the two cars from different ends of the same table coming toward each other at the same constant speed, passing each other and stopping at the beginning of the other car's starting place. Ask the subject to graph the distance versus time for these two cars.

5. Demonstrate one of the cars rolling down a ramp onto the table where it will roll until it hits a barrier. Ask the subject to draw a distance versus time graph of the motion.

6. Have students tell the story of the Tortoise and the Hare, write it, graph it, and create the scenario with the toy cars. As a challenge, have students graph both the distance versus time for the race and the graph of the speed versus time for the race with one graph above the other so that the time axes are on the same scale.

7. You may want to incorporate the iPad app "Tortoise and the Hare" for this unit.

3) <u>RESEARCH</u>: Examine/discuss research (encyclopedia entries)
Interviews with students revealed that four phases of representation lead to the most successful and powerful understanding of algebraic modeling of physical representations (Izsak, 2004). They are:

- Phase 1: Coordinating quantities and understanding distance patterns.
- Phase 2: Generating and evaluating initial algebraic representations.
- Phase 3: Coordinating understandings and revising algebraic representations.
- Phase 4: Generating further distance representations analogous to the initial problem situation. (p. 94)

Research also indicates that problems posed should be deliberately constructed to elicit modeling, and that questions be asked in successively more complex order. In this way, students may be challenged to engage with the physical apparatus so as to pose questions that they are eager to solve for their own satisfaction. This appears to be the greater contribution of using physical situations --- modeling how to pose powerful questions around a very engaging physical situation with many mathematical discoveries to be elicited from students.

4) <u>ASSESSMENT</u>: Consider assessments (Formative Assessment Database)
Using toy matchbox cars, show two cars traveling in scenarios that create ever more complex relative motion.

1. For example: Show one car moving at constant speed across a table by holding the car with your hand. Draw a distance versus time graph of the motion.

2. Add another car to the same scenario, but with a slightly greater constant speed. Draw a graph with the motion of both cars represented.

3. Create a scenario where one car has a few second head start from the second car and travels at a constant speed. But the second car travels at a higher speed than the first. Graph the distance versus time for these cars.

4. Start the two cars from different ends of the same table coming toward each other. Graph the distance versus time for these two cars.

5. Create several graphs of motion of two cars that the students will then enact with the two toy cars on a table.

6. Have students write a simple cop and speeder chase story that they represent as a graph and a scenario that they video as a demonstration. Adding numbers for distances and speeds, students should write equations that model the story as well.

7. Have students tell the story of the Tortoise and the Hare, write it, graph it, and create the scenario with the toy cars. As a challenge, have students graph both the distance versus time for the race and the graph of the speed versus time for the race with one graph above the other so that the time axes are on the same scale.

5) <u>SUGGESTIONS FOR TEACHING</u>: Consider strategies based on research (including apps)

1) Have students work in pairs solving problems
2) Use problems with physical representations that permit experimentation and manipulation
3) Pose questions carefully to move students to more sophisticated understanding of algebraic ideas
4) Permit students the time and confidence to solve the problems themselves, and also to form conjectures that they then test for themselves.
5) Develop more challenging and analogous problems to reinforce and expand student understanding.
6) When possible, look for patterns of student responses and use the patterns in the development of subsequent problem-solving tasks.
7) Ask students to describe their thoughts and problem solving strategies.
8) Encourage students to pose their own questions about the behavior and future behavior of the physical system.

6) **<u>Did the preservice teachers understand? How do you know? Evidence</u>**

<u>REFERENCES</u>

Izsak, A. (2000). Inscribing the winch: Mechanisms by which students develop knowledge structures for representing the physical world with algebra. *The Journal of the Learning Sciences, 9*, 33.

Izsak, A. (2004). Students' coordination of knowledge when learning to model physical situations. *Cognition and Instruction, 22*(1), 81-128.