BACKGROUND
This module is focused on the teaching of algebraic modeling of quantitative relationships observed in the translation of equations into verbal descriptions. The skills needed for students to make sense of equations that are used to describe quantitative relationships in the sciences are often related to their understanding of algebraic equations and expressions. This is especially critical to study in science, engineering, economics, social science, business, construction, etc.

What’s the important math?

The power of algebra is that it may be used to model quantitative relationships represented in the observed world. Students advance their understanding of both algebra and the physical world by learning to:

• Read and translate from equations into words,
• identify quantities of relevance in the equations and relate them to quantitative relationships,
• understand patterns of relationships among these quantities that are implied in the equation, and
• use the algebraic equations to make predictions, graphs, and tables of data.

1) SET: Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers’ prior knowledge)

Describe how Betty’s age relates to Sally’s age in the following equation:

\[ B - 5 = S \]

Given the following equation for temperature conversion, please describe in your own words how degrees Fahrenheit are related to degrees Celsius. Also describe why you believe it is accurate or inaccurate.

\[ F = \frac{9}{5} C + 32 \]

Given the following equation, please describe how the following equation represents, either correctly or incorrectly, the relationship between centimeters and meters:

\[ 100c = m \]
Given the following equation, please describe how the following equation represents, either correctly or incorrectly, the relationship between centimeters and meters:

\[ c = 100m \]

2) **STUDENTS:** Watch video clips of students describing their thinking as they engage with these equations and an inequality: \( x + 5 = 8; \ 2x + 5 = 8; \ 2x - 5 = 8; \) and \( 2x - 5 < 8 \).

Use these videos:

a. Boy 1 story problem
b. Girl 1 story problem
c. Girl 2 story problems

What do you learn from what you are hearing or seeing regarding students' thinking?

**Equations to Words Problems:** [note, these problems should be written one to a large index card and handed to the research subject one at a time to be read aloud by the subject. The students should be asked to write whatever equations, pictures, or expressions, tables, graphs, etc. with a permanent ink sharpie on a large flipchart. As pages get used up, they should be numbered, ripped from the pad and taped to a wall for students to refer to as needed. Newer entries should be written large and below the earlier ones in a clear succession. Students should not be permitted to erase or obliterate their work, but instead, they should place an ‘X’ next to any work that they think is mistaken or that they would like to delete.]

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\[ F = \frac{9}{5} C + 32 \]

Given the following equation, please describe how the following equation represents, either correctly or incorrectly, the relationship between centimeters and meters:

\[ 100c = m \]

Given the following equation, please describe how the following equation represents, either correctly or incorrectly, the relationship between centimeters and meters:

\[ c = 100m \]
Given the following equation please describe the relationships among the following three variables where ‘F’ stand for units of force, ‘m’ stands for units of mass, and ‘a’ stands for units of acceleration:

\[ F = ma \]

Given the following equation for the vertical height of an object thrown into the air, describe your thoughts to the following questions:

\[ H = 25t - 5t^2 \text{ where } h \text{ is the height in meters and } t \text{ is the time in seconds} \]

1) According to this equation, does the object go up forever, does it go up and come down, or is it impossible to determine from this equation alone? Describe your thoughts.
2) According to this equation, is it possible to determine how long it travels up or the amount of time that the object takes to go up and down?
3) Do you believe that an object thrown into the air takes as long for it to go up as it takes for it to come down, more time, less time? If this equation accurately predicted the motion of the object thrown, could it be used to answer the question about the amount of time needed to go up and come down? Describe your thoughts.

We usually translate from words to symbols or graphs or other “mathematical” representations. We rarely begin with the equations and translate to the narrative mode. Why is that? It seems that for students to be flexible with representations, they should have practice in this process as well.

3) RESEARCH: Examine/discuss research (encyclopedia entries) We are in the process of searching for good research sources. Have found nothing good yet!

4) ASSESSMENT: Consider assessments (Formative Assessment Database) Given the following equation please describe the relationships among the following three variables where ‘F’ stand for units of force, ‘m’ stands for units of mass, and ‘a’ stands for units of acceleration:

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4) According to this equation, does the object go up forever, does it go up and come down, or is it impossible to determine from this equation alone? Describe your thoughts.
5) According to this equation, is it possible to determine how long it travels up or the amount of time that the object takes to go up and down?
6) Do you believe that an object thrown into the air takes as long for it to go up as it takes for it to come down, more time, less time? If this equation accurately predicted the motion of the object thrown, could it be used to answer the question about the amount of time needed to go up and come down? Describe your thoughts.

5) SUGGESTIONS FOR TEACHING: Consider strategies based on research (including apps)

1) Have students work in pairs solving problems
2) Use equations with quantities that students may be familiar with
3) Pose questions carefully to move students to more sophisticated understanding of algebraic ideas
4) Permit students the time and confidence to solve the problems themselves, and also to form conjectures that they then test for themselves.
5) Develop more challenging and analogous problems to reinforce and expand student understanding.
6) When possible, look for patterns of student responses and use the patterns in the development of subsequent problem-solving tasks.
7) Ask students to describe their thoughts and problem solving strategies.
8) Encourage students to pose their own questions about the equation and how it may be used to predict future events.

6) Did the preservice teachers understand? How do you know? Evidence

REFERENCES