Center for Algebraic Thinking

MODULE

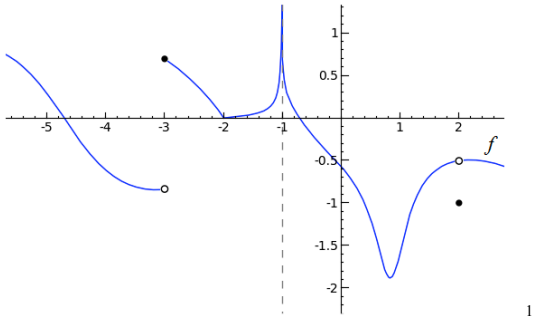
Patterns and Functions: What is a function?

BACKGROUND

The concept of function is a complex mathematical idea whose formal definition has evolved over time. This module provides an opportunity for pre-service and in-service teachers to grapple with a number of key misconceptions that often hinder students' deeper understanding of functions.

1) **SET: Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers' prior knowledge)**

- Which sketches or descriptions below represent functions?
- If one does, how do you know?
- If one does not, how could you alter the sketch or description so that it does represent a function?



[1]

[1] From http://www.sagemath.org/calctut/onesided.html

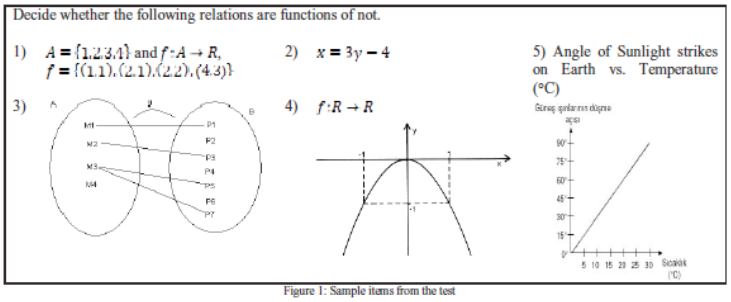


Figure 1: Sample items from the test [2]

Is there a function that is defined everywhere except for the integers?

2) **<u>STUDENTS</u>: Watch video clips of students describing their thinking as they engage with problems.**

1. "This is a function, but it's a little strange. It has jumps and an asymptote and a point off by itself, but it only has one *y* for each *x*."
Error: "This is not a function because it can't jump around."
2. "This is not a function because (2,1) and (2,2) can't both be points on the graph of a function."
Error: "This is a function because all of the points are different."
3. "This is a function. It could be *x* as a function of *y*, as it is written, but it could also be *y* as a function of *x* if I solve for *y*."
Error: "This is not a function because the *y* isn't by itself."
4. "This is not a function because the third thing gets matched with two things, and the fourth thing isn't matched at all."
Error: "This is not a function because there are things on the right that don't get hit."
Error: "This is a function because there are four arrows coming out of the four things in the domain."
5. "This is a function since each *x* is matched with just one *y*."
Error: "This is not a function because two *x*s get matched with the same *y*."
6. Error: "This is not a function since it is just a straight line."

Students should tackle the same problems as in 1). (With some modifications to facilitate access.)

3) **<u>RESEARCH</u>: Examine/discuss research (encyclopedia entries)**

The following search items in TATE may be useful in understanding the issues involved in understanding what functions are and what issues students are likely to have.

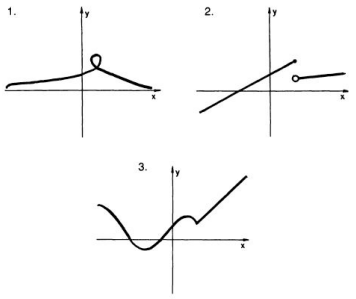
- Definition of function (Bourbaki, covariation)
- Domain and range
- Continuous vs discontinuous
- One-to-one

[2] Hatisaru, V., & Erbas, A. K. (2010), p. 3923.

- historical development of function definition

4) **ASSESSMENT: Consider assessments (formative assessment database)**

Is there a function that has the graph shown?



9/4/11 1:35 AM
Comment [1]:

Is there a function that assigns to every number different from 0 its square and to 0 it assigns 1?
Is there a function all of whose values are equal to each other?
Is there a function whose values at integers are non-integers and whose values at non-integers are integers?[3]

5) **SUGGESTIONS FOR TEACHING: Consider strategies based on research (including apps)**

If students are only exposed to polynomial functions, they may then equate the notions of "polynomial" and "function" rather than seeing polynomials as just a single class of examples of a broad concept. It is better to provide many different kinds of examples, including discontinuous functions, functions with discrete domains, functions with exceptional points, piece-wise-defined functions, verbally described functions, and functions with non-simple domains such as √(x^2-1).

The development of students' conceptions of functions mimics the historical development of the definition of function, a sort of mathematical "ontogeny recapitulates phylogeny." Teachers should be aware of and take into account this progression. For example, students do not initially consider discontinuous functions to be functions, which matches a 17th-century definition; they often consider functions to be relationships expressible by a formula, which matches an 18th-century idea of function, etc.

6) **Did the preservice teachers understand? How do you know? Evidence**

REFERENCES

Hatisaru, V., & Erbas, A. K. (2010). Students' perceptions of the concept of function: The case of Turkish students attending vocational high school on industry. *Social and Behavioral Sciences, 2*(2), 3921-3925.

Vinner, S., & Dreyfus, T. (1989). Images and definitions for the concept of function. *Journal for Research in Mathematics Education, 20*(4), 356-366.

[3]Vinner, S., & Dreyfus, T. (1989)