Center for Algebraic Thinking

MODULE

Patterns and Functions: How do examples impact our understanding of function?

BACKGROUND

Students understandings of key concepts and their ability to generalize about mathematical ideas are directly affected by the examples their teachers choose to use in the course of instruction. In this module, pre-service and in-service teachers have the opportunity to explore strategies for choosing algebraic examples that support deeper conceptual understanding for students.

1) **<u>SET</u>: Engage with a problem or problems that help teachers consider students' algebraic thinking (teachers' prior knowledge)**

Consider the following pattern:

$P_1 = 2 \times 3$

$P_2 = 3 \times 4$

$P_3 = 4 \times 5$ and so on.

What is P_{100}? What is P_n?

[2]

[1]

1. a) Give a definition of a function.
   b) A student says that he/she does not understand this definition. Give an alternate version that might help the student understand.
2. How are functions and equations related to each other ?
3. A student is asked to give an example of a graph of a function tha passes through the points A and B (See Fig. 1).
   The student gives the following answer (See Fig. 2).

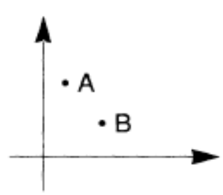

Figure 1

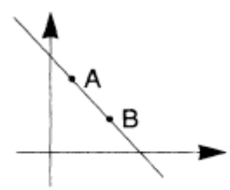

Figure 2

When asked if there is another answer the student says: "No".
- If you think the student is right—explain why.
- If you think the student is wrong—how many functions whick satisfy the condition can you find? Explain.

[1] Zazkis, R., Liljedahl, P. & Chernoff, E.J. (2008), p. 133.

Develop a definition of function that will accommodate a wide range of representations and contexts. For example, every child has a date of birth; in a classroom, the list of all children maps onto the list of their birthdays, forming a function. Make sure that your definition includes such situations as well as equations and their graphs. Consider questions such as:
- How can you know whether a graph represents a function or not?
- How can you know whether a table represents a function or not?
- How can you know whether an equation represents a function or not?

2) **<u>STUDENTS</u>: Watch video clips of students describing their thinking as they engage with problems**

Ask students to give examples of functions and discuss how functions and equations are related to each other.
Questions to ask:
Are all functions equations?
    [EX. if we map the letter of the alphabet onto the numbers 1-26, is this a function?]
Are all equations functions?
    [EX. $x^2 + y^2 = 25$]
Do students give examples that are not linear or quadratic?

[See teacher/candidate questions from Even above.]

3) **<u>RESEARCH</u>: Examine/discuss research (encyclopedia entries)**

prototypes
linear
quadratic
generalization

4) **<u>ASSESSMENT</u>: Consider assessments (formative assessment database)**

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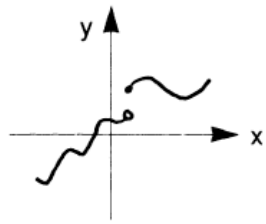
[2] Even, R. (1993). p. 99.

4. A student marked all the following as non-functions. (R is the set of all the real numbers, N is the set of all the natural numbers).

(i)



(ii) f : R —> R
f (x) = 4

(iii) g : N —> R

(iv) A correspondence that associates 1 with each positive number, –1 with each negative number, and 3 with zero.

(v) $g(x) = \begin{cases} x, & \text{if } x \text{ is a rational number} \\ 0, & \text{if } x \text{ is an irrational number.} \end{cases}$

(vi) {(1,4), (2,5), (3,9)}

a) For each case decide whether the student was right or wrong. Give reasons for each one of your decisions.

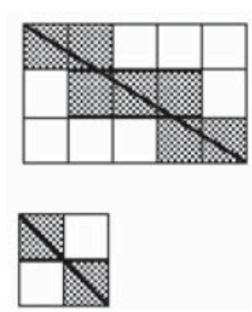
(i) Right/wrong because
(ii) Right/wrong because
(iii) Right/wrong because
(iv) Right/wrong because
(v) Right/wrong because
(vi) Right/wrong because

b) In cases where you think the student was wrong, try to explain what the student was thinking that could cause the mistake.

Diagonals in a Rectangle

On squared paper draw a rectangle and draw in a diagonal.

How many grid squares are crossed by the diagonal?



In case of a 3x5 rectangle or a 2x2 rectangle above, we can simply count.

However, can we make a decision about a 100×167 or a 3600×288 rectangle?

In general, given n×k rectangle, how many grid squares are crossed by its diagonal?

Is this a function?

5) **<u>SUGGESTIONS FOR TEACHING</u>: Consider strategies based on research (including apps)**

• Generalizing patterns is a key element of algebraic thinking. The use of "big" numbers in examples can support students' capacity to generalize; likewise, the use of "small" numbers can result in inappropriate generalizations. “Big” numbers can also help draw students’ attention to underlying structure in algebraic expressions. For example, consider these two examples of a difference of squares:

$$(2x - 3y)^2 - (x + 3y)^2 \text{ versus } (26x - 15y)^2 - (24x + 15y)^2$$

Studies indicate that students indicated a deeper appreciation for this underlying structure when the coefficients were not necessarily “big,” but simply “bigger.”

• Similarly, the prevalence of the linear function as the prototypical function example leads students to use the properties of linear functions to generalize rather than the definition of the function concept.

• It is important that teachers choose examples beyond linear and quadratic functions (such as - polynomial, absolute value, greatest integer, etc.) so that students expand their function concept image to include a wide range of function types.

• Teachers

• Students should given an opportunity to experiment with graphing software (for example “Grapher” from the National Library of Virtual Manipulatives) and to form conjectures about the relationships between the following sorts of algebraic transformations:

> **stretches -** $af(x)$ and $f(ax)$
> **shifts -** $f(x + k)$
> **sums -** $f(x) + g(x)$
> **differences -** $f(x) - g(x)$
> **products -** $f(x) \cdot g(x)$

6) Did the preservice teachers understand? How do you know? Evidence

REFERENCES

Even, R. (1993). Subject-Matter Knowledge and Pedagogical Content Knowledge: Prospective Secondary Teachers and the Function Concept. Journal for Research in Mathematics Education, 24(2), 94-116.

Schwarz, B. B., & Hershkowitz, R. (1999). Prototypes: Brakes or Levers in Learning the Function Concept? The Role of Computer Tools. *Journal for Research in Mathematics Education*, 30(4), 362-389.

Zazkis, R., Liljedahl, P. & Chernoff, E.J. (2008). The role of examples in forming and refuting generalizations. *ZDM Mathematics Education*, 40:131-141.