Center for Algebraic Thinking

MODULE

Algebraic Relations: Informal Procedures and Student Intuition

BACKGROUND

When solving equations, students often resort to applying taught algorithms to find the answer. However, when students use their intuition and informal procedures they can learn to move flexibly between solution strategies. Students can also avoid errors that sometimes come in problems with a large number of steps.

1) **SET: Engagement with a problem or problems that help teachers to consider students' algebraic thinking.**

Task	Instruction	Discussion
$\frac{x}{4}+6=10$	Solve for x.	How did you solve for x?
$\frac{5x}{2}-15=5$	Solve for x without using a pencil.	How did you solve for x?
Create a task that for your table partner to solve without using a pencil.		What are the features of a problem that can be solved without a pencil?

In the first task, most pre-service teachers will start by subtracting 6 from both sides and then multiplying both sides by 4 to find that x=16. An alternate strategy is to think through the problem by asking yourself, "What added to six would equal 10? 4! That means that $\frac{x}{4}$=4. So, what divided by 4 is 4? X must equal 16."

This task is a good example of how students can use their own reasoning to solve equations. Because they have been taught formal procedures for solving equations, student abandon their own intuition and reasoning skills about what the solutions might be. Students cling to these solutions strategies even when applying them incorrectly generates solutions that could not be reasonable.

2) **STUDENTS: Watch video of students describing their thinking as they engage with problems.**

What do you learn from what you are hearing or seeing regarding students' thinking?

Why do you think the student used a different strategy for the box problem?

How can we press the student into using reasoning to solve the problem?

3) **<u>RESEARCH</u>: Examine/discuss research (encyclopedia entries)**
See the Encyclopedia entry titled, "Informal Procedures and Student Intuition."
See the Encyclopedia entry titled, "Linear Equations."
See the Encyclopedia entry titled, "Flexible Use of Solution Strategies."
Read (Kuchemann, 1983) for more information

4) **<u>ASSESSMENT</u>: Consider assessments (formative assessments)**

Challenge your students to think about the features of tasks that can be simplified through reasoning, rather than procedures. For each of the following tasks, decide which strategies would be most helpful: reasoning, formal procedures, or both. Be prepared to justify you decisions.

Task	Solve for x. Use your reasoning!
1.	$3(x+5)=21$
2.	$10-x=6$
3.	$3(x+5)=20$
4.	$\frac{24}{x}=12$
5.	$2x-5=15$
6.	$2x=x+6$
7.	$\frac{10}{x+9}=1$
8.	$\frac{1}{2}(x+3)=6$
9.	$9-2x=x$
10.	$\frac{x+2}{5}=2$
11.	$3(x+2)+x=3(x+2)-1$
12.	$2x+1-9=31-9$
13.	$\frac{4}{a}=3$

Tasks #1, #2, #4, #5, #6, and #10 are fairly easy to solve without the use of formal procedures. Tasks #3, #9 and #13 are more easily solved by using formal procedures. Tasks #7, #8, and #11 are clearly easier to solve without formal procedures. What are the features of a task that is more efficiently solved with reasoning? What are the features of a task that is more efficiently solved with formal procedures?

It is important to give students the choice to use intuition OR formal procedures whenever they solve a task. We do not want to make "intuition" into another procedure that we require. Instead, our goal is for students to move flexibly between solution strategies (Star & Rittle-Johnson, 2007) using whichever strategy gives them an efficient and accurate solution.

5) **<u>SUGGESTIONS FOR TEACHING</u>: Consider strategies based on research (including apps)**
One strategy for developing intuition in solving equations is through the use of the "cover up" method (Kuchemann, 1983). Students look at equations and use their fingers to "cover up" messy parts so that they can think more clearly about the problem. For example, in Task #8, a student might place their finger over the parenthesis and ask themselves, "One half of what is 6? So, whatever is under my finger must be 12. So x + 3 is 12. What plus 3 gives me 12? X must be 9."

The cover up method is not always the most efficient way to solve an equation (see Tasks #3, #9, and #12). When students discover this, we create a situation where "showing your steps" and using procedures is desirable. This can be a powerful method for demonstrating the value in showing steps and using procedures.

One tool for differentiating between tasks solvable by intuition and tasks requiring formal procedures is the Cover Up app. Students could be directed to solve a set of equations using this app. It will be critical for students to distinguish between the tasks solved using intuition and the tasks solved using formal procedures.

6) **<u>Did the preservice teachers understand? How do you know? Evidence</u>**

<u>REFERENCES</u>

Kuchemann, D. (1983). Quantitative and Formal Methods for Solving Equations.Mathematics in School, 12(5), 17-19.

Star, J. R., & Rittle-Johnson, B. (2008). Flexibility in Problem Solving: The Case of Equation Solving. Learning and Instruction, 18(6), 565-579.