



Center for Algebraic Thinking

MODULE

Algebraic Relations: The Meaning of the Equal Sign

BACKGROUND

The goal of this module is to help pre-service teachers to develop their understanding of “=” and help them to understand the relational properties of “=” and highlight the difference between operators symbol such as “+”, “-”, “/”, “...”, and “=”

1) SET: Engagement with a problem or problems that help teachers consider students' algebraic thinking.

Question 1:

Your student solves the problem: $235 - 65$ as:

$$\begin{aligned} 235 - 5 &= 230 - 30 \\ &= 200 - 30 \\ &= 170 \end{aligned}$$

Is this correct? If not, what is wrong? What is a question that we could ask the student to highlight the error?

Question 2(Hattikudur & Alibali, 2010):

One of your students answered the question: $36 + 14 = \square + 5$ as $36+14 = 50+5$.

Is this correct? If not, what is wrong? What is a question that we could ask the student to highlight the error?

Are the errors in Questions 1 and 2 the same? What are the differences/similarities?

List examples of appropriate use of equal sign. Is = an operator like addition, subtraction, division and multiplication? How might “calculator use of equal sign” reinforce misconceptions of = ? Why might some calculators (like Texas Instruments) have an “Enter” button instead of = ? Is there a benefit to this?

Question 3 (Godfrey & Thomas, 2008):

Are these uses of = appropriate? Are they similar to the examples in your list?

a) $100=100$

- b) $12 - 8 = 4$
- c) $12 \times 3 = 6 \times 6$
- d) $60 \div 5 = 3 \times 4$
- e) $12 + 9 = 3 \times 7 = 63 \div 3$
- f) $12 + 9 = 21 + 9 = 30$
- g) $18 - 10 = 8 = 2 \times 4$
- h) $25 - 6 + 11 = 19 + 11 = 30$

Question 4 (Godfrey & Thomas, 2008):

What if we have expressions with variables?

- i) $w = w$
- j) $2w = w$
- k) $k = 5$
- l) $5r - 1 = 11$
- m) $5t - t = 4t$
- n) $3w = 7w - 4w$
- o) $2w = 7w - 6w$
- p) $w = w + 1$

These tasks are examples for pre-service teachers to discuss the meaning of equal sign as a symbol, and how it is different than other operators. Research indicates that K-12 students use equal sign as an operator symbol to mean “the answer is” or “find the answer”. The questions in this set plan to help pre-service teachers to understand that $=$ is a symbol to indicate a special relationship between two sets of expressions.

2) **STUDENTS: Watch video of students describing their thinking as they engage with problems.**

What do you learn from what you are hearing or seeing regarding students' thinking?

3) **RESEARCH: Examine/discuss research (encyclopedia entries)**

See Encyclopedia entries on “Equal Sign”.

See also “Inequality” entries for additional information on inequalities.

4) **ASSESSMENT: Consider assessments (formative assessments)**

The following problems could also be used in K-12 classes.

(1) Challenge your students to think about the meaning of equal sign through the following questions. For each of the following tasks, decide if $=$ is used appropriately. If not, decide on an appropriate sign.

- a. $6 = 6$
- b. $12 - 8 = 4$
- c. $2 = 10 \div 5$
- d. $6 = 2 \times 5 - 3$
- e. $3 = 5 - 4$
- f. $7 \times 2 = 2 \times 7$
- g. $19 + (11 + 8) = (19 + 11) + 8$
- h. $6 \times 8 + 11 = 48 + 11 = 59$

- i. $7 \times 2 + 11 = 14 + 11 + 87 = 113$
- j. $87 + 13 = 74 - 5 = 69$

Parts in Question 1 helps students to understand the use of = with expressions with numbers only and also provides correct and incorrect use of it when more than two sets of expressions are compared. This question also helps students to compare two sets of expressions with numbers and promotes the use of >, < and ≠ by comparing numbers.

(2) Challenge your students to think about the meaning of equal sign through in the following statements. For each of the following tasks, decide if = is used appropriately. Discuss if there would be conditions under which = could be used or it is more appropriate to use other symbols.

- a. $t = t$
- b. $2t = t$
- c. $a + b = b + a$
- d. $2w + (3 + 5x) = (2w + 3) + 5x$
- e. $x(x + 2) = x^2 + 2x$
- f. $8x - 6x = 2x$
- g. $8x - 5x = 2x$
- h. $0 = x^2 + 2x - 5$
- i. $2w + 1 = 2w$

Question 2 provides an opportunity to discuss the meaning of equal sign more in depth using expression with variables.

5) SUGGESTIONS FOR TEACHING: Consider strategies based on research

Ask students to put the correct sign. Imagine a student makes an error. What is a question that you could ask that student to cause him/her to see the error?

Expression 1	Sign: =, >, <	Expression 2	Error	Question to ask the student:
$47 - 11$		33	$<$	
9×3		$27 - 11$	$=$	
$18 \div 6 + 1$		3	$<$	
$6 \times 8 + 11$		$48 + 11$	$=$	

6) Did the preservice teachers understand? How do you know? Evidence

REFERENCES:

Godfrey, David, & Thomas, Michael O. J. (2008). Student Perspectives on Equation: The Transition from School to University. *Mathematics Education Research Journal*, 20(2), 71-92.

Hattikudur, S., & Alibali, M. W. (2010). Learning about the equal sign: Does comparing with inequality symbols help? *Journal of Experimental Child Psychology*, 107, 15-30.